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N87-16783 26P
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1986

NASA/ASEE SUMMER FACULTY RESEARCH FELLOWSHIP PROGRAM

**MARSHALL SPACE FLIGHT CENTER
THE UNIVERSITY OF ALABAMA**

**SOME ATMOSPHERIC SCATTERING CONSIDERATIONS RELEVANT
TO BATSE: A MODEL CALCULATION**

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Date:	August 15, 1986
Contract Number:	NGT 01-002-099 The University of Alabama

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ABSTRACT

The orbiting Burst and Transient Source Experiment (BATSE) will locate gamma ray burst sources by analysis of the relative numbers of photons coming directly from a source and entering its prescribed array of detectors. In order to accurately locate burst sources it is thus necessary to identify and correct for any counts contributed by events other than direct entry by a mainstream photon. The present work describes an effort to estimate the photon numbers which might be scattered into the BATSE detectors from interactions with the Earth's atmosphere. A model has been developed which yielded analytical expressions for single-scatter photon contributions in terms of source and satellite locations.

ACKNOWLEDGEMENTS

I would here like to express my sincere feelings of gratitude for having been given the opportunity to participate in the NASA/ASEE Summer Faculty Fellowship Program. It has provided an experience of challenge and satisfaction and I hope the program will long continue to the benefit of participants and NASA.

My colleagues of the past two years, Drs. Gerald Fishman and Charles Meegan, deserve a special thanks for graciously accepting my participation into a research area with which I had relatively little familiarity. I hope that my efforts will prove useful to their program.

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INTRODUCTION

One of four experiment packages aboard the Gamma Ray Observatory (GRO) is the Burst Transient Source Experiment (BATSE). Utilizing an array of eight NaI detectors, BATSE will make angular location determinations of the gamma ray burst sources by means of analyzing the relative count responses of its detectors. Figure 1 depicts the GRO configuration¹. If the planes of the detector faces were extended so as to form a closed surface, that surface would be the regular octohedron also depicted in Figure 1.

The assignment of a numbering scheme to the detectors as indicated in Figure 2 can be used to determine the angular coordinates, (θ, ϕ) , of a burst as follows. Suppose that, for a given burst, detectors 1, 2, and 3 are those presumed to optimally locate the burst source. Having recorded respective count rates N_1 , N_2 , and N_3 allows the (uncorrected for error sources,² or "zeroth-order") angular coordinates to be computed from²

$$\tan \theta^* = [N_1^2 + N_2^2 + N_3^2 + 2N_3(N_2 - N_1)]^{1/2}, \quad (1-a)$$

$$\tan \phi^* = (N_2 + N_3)/(N_1 - N_3). \quad (1-b)$$

The tacit assumption made of the N_i appearing in these expressions is that they are directly from the source (i.e., no scattering has contributed) and that the BATSE detectors are perfect. Any error source in the N_i will, of course, give rise to uncertainties, $\delta\theta$ and $\delta\phi$, in the source location. Thus, in utilizing counts to make the source location determinations, it is imperative to identify all possible sources of error in the recorded counts and to compensate for these in the location calculations.

GRO will orbit at an altitude of some 400 km., well above the Earth's atmosphere. It might be anticipated that, in the course of a given burst, a significant number of gamma rays might scatter from the Earth's atmosphere and enter the Earth-facing BATSE detectors. Such contributions would obviously produce misleading data and corrections for the scattered contribution must be made. An extensive study of the atmospheric scattering problem has been previously

undertaken by Moris³ using Monte Carlo calculations to examine probable photon interactions with the atmosphere. The results of that study were presented in graphical form in a way indicating the relationship of the ratio of observed-to-incident flux from a given zenith and satellite location as a function of gamma ray energy.

The objective of the present work is to develop a model for the scattering contribution from which analytical expressions for the anticipated scatter contributions can be derived. The contributions would then be used to correct Equations (1) and, hopefully, lead to a refinement in the source location computation. In addition, analytical expressions offer the advantage of presenting statements which can be more readily interpreted than numerical results; it is thus hoped that this work can shed further light on the difficult scattering problem.

GAMMA RAY INTERACTIONS WITH THE ATMOSPHERE

Gamma rays entering the Earth's atmosphere may be expected to interact with the electrons of that medium (bound primarily to nitrogen and oxygen, whose abundances are roughly 80% and 20%, respectively). The three principal interactions will be photoelectric absorption, Rayleigh scattering, and Compton scattering. The first of these will serve to attenuate the burst intensity as it propagates through the atmosphere, while the two types of scattering processes produce direction changes, along with an energy degradation in the case of Compton scattering.

Suppose a gamma ray enters the atmosphere between the GRO orbit and the Earth's surface as depicted in Figure 3. Consider that portion of the incident flux, I_0 , which propagates through a column of cross sectional area $A = 1 \text{ cm}^2$ and depth d , at which point scattering finally occurs. A typical photon energy in the range of interest in the BATSE may be taken as 100 keV. Relative to this energy electron binding energies are small⁴ and the scattering electrons may be approximated as being free; thus the scattering may be regarded as Compton, with total cross section σ_c . Since the total cross section represents the probability of a scattering event per electron, the photon must encounter n_e electrons before scattering is a certainty: $n_e \sigma_c \equiv 1$, meaning that $1/\sigma_c$ electrons must be encountered by a photon in the column of Figure 3 before scattering is certain to occur. If the atmosphere is regarded as being composed of two types of atoms, a and b, with densities $n_a(\vec{r})$ and $n_b(\vec{r})$ the number of electrons within the column will be

$$n_e = Z_a \int n_a(\vec{r}) dv + Z_b \int n_b(\vec{r}) dv, \quad (2)$$

Z being the atomic number. Equating this to $1/\sigma_c$ in essence gives a numerical value to the integrals since the Compton cross section (as a function of energy) is well known.

Next the photoelectric attenuation over this column depth must be evaluated. The intensity diminishes by dI along some infinitesimal path length $d\xi$ of the column according to

$$dI = - \mu I d\xi , \quad (3)$$

where μ is the attenuation coefficient and is related to the electron population, Zn , and absorption cross section, σ , in product form.⁵ With variable densities of two types γ and σ , here representing the photoelectric cross section, the intensity diminishes as

$$dI = - I [Z_a n_a(\vec{r}) + Z_b n_b(\vec{r})] \sigma_\gamma d\xi \quad (4)$$

Multiplication and division of the right hand side by A gives

$$I = I_0 \exp \{- \sigma_\gamma [Z_a \int n_a(\vec{r}) dv + Z_b \int n_b(\vec{r}) dv]\} . \quad (5)$$

The integral part of the argument is just n_e , that is $1/\sigma_c$, giving the attenuation of the incoming beam along the scattering path as

$$I = I_0 \exp [- \sigma_\gamma / \sigma_c] . \quad (6)$$

If the scattered beam is imagined to undergo the same magnitude of attenuation in its (outward) propagation, the inclusion of a factor of 2 in the argument of Equation (6) will relate the attenuated intensity which might reach GRO in terms of the incident intensity undergoing a single scattering event. The specific form of the energy-dependent absorption cross section is given by

$$\sigma_\gamma = \frac{4 \sqrt{2}}{(137)^4} \frac{Z^5 \sigma_0}{\alpha^{7/2}} \quad (7)$$

where⁶ Z here will be taken as an average for the 80% nitrogen ($Z = 7$) and 20% oxygen ($Z = 8$) composition, σ_0 is the Thomson cross section ($= 6.65 \times 10^{-25} \text{ cm}^2$) and α is the

ratio of photon energy to the electron rest energy. The energy-dependent Compton cross section can be expressed as⁸

$$\sigma_c = f(\epsilon) \sigma_0, \quad (8)$$

where $f(\epsilon)$ is a decreasing function of energy. Finally, putting together these results relates the (single) scattered intensity that might reach GRO and the incident intensity as

$$I = I_0 \exp \{ - (6.2 \times 10^{-4}) / f(\epsilon) \epsilon^{7/2} \}. \quad (9)$$

Elementary considerations⁹ of a scattering situation in which a beam of intensity I_0 is incident normally onto a thin target material anticipate that a detector some distance r from a scattering center will receive flux scattered between θ and $\theta + d\theta$ (measured from the incident beam direction) given by

$$I' = \frac{I_0}{r^2} \left(\frac{d\sigma}{d\Omega} \right), \quad (10)$$

where $d\sigma/d\Omega$ is the differential scattering cross section (per electron) for the particular scattering event. Scattering occurs symmetrically about the incident beam direction and it should be noted here that if the beam were oriented at some angle ζ with respect to the scattering surface, the normal component would be $I_0 \cos \zeta$, and the $\cos \zeta$ factor would be included on the right hand side of Equation (10).

Since the BATSE detectors count photon numbers rather than energies it is convenient to describe the scattering on terms of numbers. If a certain portion, dI_0 , of the incident flux is due to dN_0 photons at energy ϵ_0 , $dI_0 = dN_0 \epsilon_0$. Similarly, if dN' is the portion of the total number scattering at energy ϵ' , $dI' = dN' \epsilon'$. Equation (10) can then be expressed, allowing for non-normal incidence, as

$$dN' = \frac{dN_0 \cos \zeta}{r^2} \left(\frac{\epsilon_0}{\epsilon'} \right) \left(\frac{d\sigma}{d\Omega} \right). \quad (11)$$

The two types of scattering of interest here are, as previously mentioned, are Rayleigh and Compton. Rayleigh scattering is elastic, while in the Compton case the ratio of energy of the incident to scattered photon energy can be simply expressed in terms of the scattering angle:

$$\frac{\epsilon_0}{\epsilon'} = 1 + \alpha(1 - \cos \theta) . \quad (12)$$

Subscripting the θ -dependent differential scattering cross sections of the two types for purposes of distinction gives then

$$dN' = \frac{dN_0 \cos \tau}{r^2} \left[\left(\frac{d\sigma(\theta)}{d\Omega} \right)_R + \left(\frac{\epsilon_0}{\epsilon'} \right) \left(\frac{d\sigma(\theta)}{d\Omega} \right)_C \right] , \quad (13)$$

where the energy ratio is given by Equation (12).

The detailed forms of the differential scattering cross sections depend, of course, upon the specifics of the interactive processes operative in the particular scattering event under consideration. The forms appearing in Equation (13) are given by¹⁰

$$\left(\frac{d\sigma}{d\Omega} \right)_R = (1.3 \times 10^{-8}) \frac{z^3 \sigma_0}{\alpha^3} \left(\frac{1 + \cos^2 \theta}{2 \sin^3 \frac{\theta}{2}} \right) \quad (14)$$

and¹¹

$$\left(\frac{d\sigma}{d\Omega} \right)_C = \frac{r_0^2 (1 + \cos^2 \theta)}{2 [1 + \alpha(1 - \cos \theta)]^3} \left\{ 1 + \frac{\alpha^2 (1 - \cos \theta)^2}{(1 + \cos^2 \theta) [1 + \alpha(1 - \cos \theta)]} \right\} . \quad (15)$$

Equation (15) is the Klein-Nishina formula and $r_0 = (3\sigma_0/4\pi)^{1/2}$ is referred to as the "classical electron radius" whose value is⁷ 2.82×10^{-13} cm.

THE SINGLE SCATTER MODEL

Atmospheric scattering effects on the OSO5 hard x-ray experiment have been discussed in some detail by Langer et al.¹² Their analysis indicated that, given the atmospheric composition¹³, 30 keV photons should scatter at a height of about 40 km above the Earth's surface, while 500 keV photons scatter near 30 km. The energy range of interest to BATSE (maximum detector efficiency range) is, roughly, 25 keV to 250 keV. The above scatter altitudes are thus relevant to the present study and their slight difference in comparison with the 400 km GRO orbit suggests representing the scattering as taking place from a sphere of radius ~40 km larger than the Earth's radius.

Limiting the present study to single scatter contributions, all of which are imagined to come from a spherical surface of radius about 6411 km, means that any photons scattering into the BATSE detectors must have scattered from a point within the spherical cap depicted in Figure 4. The scattering region (cap) is defined by imaging a line extending from GRO tangent to the scattering sphere. Allowing that tangent to sweep around the vertical from GRO to the sphere's surface will define a cone whose intersection with the scattering sphere gives the region from which single scatter events might effect BATSE data. That GRO is at altitude 360 km above the scattering region means that the cap might be reasonably approximated as a flat disc.

Consider then the situation shown in Figure 5-a. N_0 photons (per cm^2 per sec) are imagined incident onto a plane at some angle ζ . The spatially uniform flux over a disc of area A_D will give rise to $dN_0 \cos \zeta$ striking the area of the washer of radii r' and $r' + dr'$; dN_0 can be expressed in terms of N_0 as

$$dN_0 = N_0 \left(\frac{2\pi r' dr'}{A_D} \right) . \quad (16)$$

All points within the washer are equidistant, r , from GRO which remains at altitude h above the plane. As previously mentioned scattering will be axially symmetric about the incident axis. Thus, only that small fraction of photons scattered through the required θ and into $\Delta\phi/2\pi$ can contribute to detector counts. But $\Delta\phi$ is the axial angle subtended by GRO dimensions, $\Delta\phi = \Delta w/r$, and this must

multiply the dN' in order to obtain the possible number of photons scattered into the GRO direction.

The above results are now to be assembled in order to compute the anticipated numbers of photons scattered into the BATSE detectors. The number possibly scattered into GRO is

$$dN'_{\text{GRO}} = \frac{\Delta W}{2\pi r} dN' \quad (17)$$

dN' is that number scattered (per electron) and is computed by using Equation (16) in Equation (13) with the attenuation factor given in Equation (6) multiplying (i.e., diminishing) the incident number of photons per cm^2 per sec. These steps give

$$dN'_{\text{GRO}} = N_0 \cos \zeta \Delta W e^{-(\sigma_Y/\sigma_C)} \left(\frac{r' dr'}{r^3 A_D} \right) \left[\left(\frac{d\sigma}{d\Omega} \right)_R + \left(\frac{\epsilon_0}{\epsilon^r} \right) \left(\frac{d\sigma}{d\Omega} \right)_C \right] \quad (18)$$

That the differential scattering cross sections refer to scattering per electron means that dN'_{GRO} must finally be multiplied by the total number of scattering electrons imagined to reside on the scattering disc. Assuming the uniform distribution $\delta \text{ cm}^{-2}$ over A_D means that δA_D scattering electrons reside on the disc. The total number of photons that can scatter into GRO is the product of δA_D and Equation (18):

$$dN'_{\text{GRO TOT}} = N_0 \cos \zeta \delta A_D e^{-(\sigma_Y/\sigma_C)} \left[\left(\frac{d\sigma}{d\Omega} \right)_R + \left(\frac{\epsilon_0}{\epsilon^r} \right) \left(\frac{d\sigma}{d\Omega} \right)_C \right] \frac{r' dr'}{r^3}. \quad (19)$$

The relevant geometry of Figure 5-a has been highlighted in Figure 5-b. It is evident that

$$\tan \phi = \frac{r'}{h}; \quad (20)$$

further, since

$$\theta + \phi + \zeta = \pi , \quad (21)$$

it follows that

$$\frac{r'}{h} = - \tan (\theta + \zeta) . \quad (22)$$

In order that $r'/h > 0$ it is apparent that $\frac{\pi}{2} < (\theta + \zeta) < \pi$. Equation (21) is equivalent to

$$\theta = \pi - \zeta - \phi$$

θ_{\max} corresponds to the minimum value of ϕ while θ_{\min} occurs when ϕ takes on its maximum value. The minimum value of ϕ is obviously zero, while the maximum occurs when r' takes on its maximum value. The latter is simply the radius of the scattering disc which examination of Figure 4 shows to be $R_A \sin(18.5^\circ)$, R_A being the 6411 km radius of the scattering sphere. Thus,

$$\theta_{\max} = \pi - \zeta; \quad \theta_{\min} = \pi - \zeta - \tan^{-1} \left[\frac{R_A \sin(18.5^\circ)}{h} \right] . \quad (23)$$

It is a simple matter now to convert the integration variable of Equation (19) to the scattering angle. It follows immediately from Equation (22), along with the Pythagorean relationship of r and r' that

$$\frac{r' dr'}{r^3} = \frac{1}{h} \sin (\theta + \zeta) d\theta . \quad (24)$$

Substitution of Equation (24) into (19) gives

$$\frac{dN'_{\text{GRO}}}{\text{TOT}} = \frac{N_0 \delta \Delta W}{h} e^{-(\sigma_Y/\sigma_C)} \left\{ \cos^2 \zeta \left[\left(\frac{d\sigma}{d\Omega} \right)_R \sin \theta d\theta + \left(\frac{\epsilon_0}{\epsilon'} \right) \left(\frac{d\sigma}{d\Omega} \right)_C \sin \theta d\theta \right] \right. \\ \left. + \cos \zeta \sin \zeta \left[\left(\frac{d\sigma}{d\Omega} \right)_R \cos \theta d\theta + \left(\frac{\epsilon_0}{\epsilon'} \right) \left(\frac{d\sigma}{d\Omega} \right)_C \cos \theta d\theta \right] \right\} \quad (25)$$

Evaluation of the four integrals arising from Equation (25) is necessary to now complete the assessment of the number of photons that might be expected to scatter into the BATSE detectors. The integrals are:

$$\int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{d\sigma}{d\Omega} \right)_R \sin \theta d\theta = (1.3 \times 10^{-8}) \frac{\sigma_0 Z^3}{\alpha^3} \int_{\theta_{\min}}^{\theta_{\max}} \frac{1 + \cos^2 \theta}{2 \sin^3 \frac{\theta}{2}} \sin \theta d\theta \quad (26)$$

$$\int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{d\sigma}{d\Omega} \right)_R \cos \theta d\theta = (1.3 \times 10^{-8}) \frac{\sigma_0 Z^2}{\alpha^3} \int_{\theta_{\min}}^{\theta_{\max}} \frac{1 + \cos^2 \theta}{2 \sin^3 \frac{\theta}{2}} \cos \theta d\theta \quad (27)$$

$$\int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{\epsilon_0}{\epsilon'} \right) \left(\frac{d\sigma}{d\Omega} \right)_C \sin \theta d\theta = \frac{3\sigma_0}{16\pi} \int_{\theta_{\min}}^{\theta_{\max}} \left\{ \frac{1 + \cos^2 \theta}{[1 + \alpha(1 - \cos \theta)]^2} \right. \\ \left. + \frac{\alpha^2 (1 - \cos \theta)^2}{[1 + \alpha(1 - \cos \theta)]^3} \right\} \cos \theta d\theta \quad (28)$$

$$\int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{\epsilon_0}{\epsilon'} \right) \left(\frac{d\sigma}{d\Omega} \right)_C \cos \theta d\theta = \frac{3\sigma_0}{16\pi} \int_{\theta_{\min}}^{\theta_{\max}} \left\{ \frac{1 + \cos^2 \theta}{[1 + \alpha(1 - \cos \theta)]^2} \right. \\ \left. + \frac{\alpha^2 (1 - \cos \theta)^2}{[1 + \alpha(1 - \cos \theta)]^3} \right\} \cos \theta d\theta \quad (29)$$

The integrals appearing in Equations (26), (27), and (28) are easily reduced to algebraic forms, while that in Equation (29) is not so simple. The results are the following:

$$\int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{d\sigma}{d\Omega} \right)_R \sin\theta d\theta = \sqrt{2} (1.3 \times 10^{-8}) \frac{\sigma_0 Z^3}{\alpha^3} \left[\frac{4}{\sqrt{1-\cos\theta}} + 4\sqrt{1-\cos\theta} - \frac{2}{3}(1-\cos\theta)^{3/2} \right]_{\theta_{\min}}^{\theta_{\max}} \quad (30)$$

$$\int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{d\sigma}{d\Omega} \right)_R \cos\theta d\theta = \sqrt{2} (1.3 \times 10^{-8}) \frac{\sigma_0 Z^3}{\alpha^3} \left\{ \frac{7}{2\sqrt{2}} \ln \left[\frac{\sqrt{1-\cos\theta}-2}{\sqrt{1+\cos\theta}+2} \right] - \sqrt{1+\cos\theta} \left[\left(4 + \frac{2}{3}\cos\theta \right) + \frac{1}{1-\cos\theta} \right] \right\}_{\theta_{\min}}^{\theta_{\max}} \quad (31)$$

$$\int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{\epsilon_0}{\epsilon'} \right) \left(\frac{d\sigma}{d\Omega} \right)_c \sin\theta d\theta = \frac{3\sigma_0}{16\pi} \left\{ \frac{1}{\alpha^2} (1-\cos\theta) - \frac{(2\alpha^2+2\alpha+1)}{\alpha^3 [1+\alpha(1-\cos\theta)]} + \frac{2(1-\cos\theta) + \frac{3}{2\alpha}}{[1+\alpha(1-\cos\theta)]^2} - \frac{1}{\alpha^2} (1+2\alpha) \ln [1+\alpha(1-\cos\theta)] \right\}_{\theta_{\min}}^{\theta_{\max}} \quad (32)$$

$$\begin{aligned}
\int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{\epsilon_0}{\epsilon} \right) \left(\frac{d\alpha}{d\Omega} \right)_c \cos\theta d\theta &= \frac{3\sigma_0}{16\pi} \left\{ \frac{1}{\alpha^3} (2+2\alpha - \alpha^2)\theta + \frac{1}{\alpha^2} \sin\theta \right. \\
&+ \frac{\alpha(1+\alpha)(1+\cos\theta)\sin\theta}{2(1+2\alpha)^2 [1+\alpha(1-\cos\theta)]^2} \\
&+ \frac{(11\alpha^5 + 11\alpha^4 - 30\alpha^3 - 48\alpha^2 - 24\alpha - 4)\sin\theta}{2\alpha^3 (1+2\alpha)^2 [1+\alpha(1-\cos\theta)]} \quad (33) \\
&+ \left. \frac{(11\alpha^5 + 11\alpha^4 - 48\alpha^2 - 24\alpha - 24\alpha - 4)}{\alpha^3 (\sqrt{1+2\alpha})^5} \tan^{-1}(\sqrt{1+2\alpha} \tan \frac{\theta}{2}) \right\}_{\theta_{\min}}^{\theta_{\max}}
\end{aligned}$$

Evaluation of these results over the limits given in Equation (23) gives, upon multiplication by the factors indicated in Equation (25), the integrated form of Equation (25) which represents the anticipated number of photons scattered into the detector direction. It is apparent that the Rayleigh contribution is wholly ignorable.

The angle ζ will not be known in general and so must be expressed in terms of more appropriate quantities, namely satellite location angles, (θ_0, ϕ_0) , and burst location coordinates, (θ, ϕ) , computed from Equation (1). Reference to Figure 6 shows that ζ is defined locally as the angle between the burst direction and the position vector of GRO. Represent, for the moment, the burst location in terms of geocentric angular coordinates (θ_Y, ϕ_Y) . The incoming burst flux can then be represented as

$$\hat{N}_O^Y = -N_O^Y (\sin\theta_Y \cos\phi_Y \hat{i} + \sin\theta_Y \sin\phi_Y \hat{j} + \cos\theta_Y \hat{k}) \quad (34)$$

Let \hat{R}_O be the GRO location at this time:

$$\hat{R}_O = R_O (\sin\theta_0 \cos\phi_0 \hat{i} + \sin\theta_0 \sin\phi_0 \hat{j} + \cos\theta_0 \hat{k}). \quad (35)$$

The angle ζ can easily be seen as

$$\cos \zeta = \left(\frac{\vec{R}_O}{R} \right) \cdot \left(\frac{-\vec{N}_O^Y}{N_Y} \right) = \cos \zeta (\theta_Y, \phi_Y; \theta_O, \phi_O) \quad (36)$$

The burst location, however, is made not in terms of (θ_Y, ϕ_Y) but in terms of (θ, ϕ) . Use of the known orientation of the satellite axes relative to the geocentric axes enables θ_Y and ϕ_Y to be expressed in terms of calculable functions of θ and ϕ . Use of the latter transformations give $\cos \zeta = \cos \zeta(\theta, \phi; \theta_O, \phi_O)$ and expressing ζ in terms of these coordinates in the integrated form of Equation (25) finally gives the scattered contribution to GRO in terms of burst location and satellite coordinates.

DISCUSSION

The objective of the present work was to develop analytical expressions for estimating the atmospheric scattering contribution to detected photon counts in the course of a burst event. Having such expressions in terms of source and satellite location coordinates would, presumably, furnish a more descriptive picture of the scattering effects than would be offered by purely numerical results. Although the objective seems to have been achieved in principle, by means of a model calculation, the cumbersome nature of the final expressions does not allow a quick and easy grasp of the picture to be made. Their most important contribution might be made by their use in examining various important cases not covered in the Morris³ study. To this date only limited numerical evaluation of the results have been carried out. In particular the shape of the back-scatter curve as a function of photon energy for $\zeta = 0^\circ$ and an infinite disc (an approximation which enormously simplifies evaluation over the limits) has been investigated and found to be in good agreement with the Morris-type curves. Inputting the final forms to a computing facility should produce curves for any desired situation.

The flat disc model used here was, obviously, for ease in calculation. It indicated that such calculations are indeed possible and so a redevelopment of the results for a spherical cap should be manageable. The fact that, at a 400 km orbital altitude, GRO will see only about 2.75% of the Earth's surface seems to make the disc model reasonable, but not perfect. In particular, a spherical cap could offer a "shadow region" to a burst, as the curvature would hide that part of the scattering region away from the burst direction; the flat disc, of course, offers no such feature and thus might suggest some additional contributions to scattering which would not actually occur.

The model discussed also considers only single scatter contributions. The Rayleigh contribution, as already noted, is completely negligible for photon energies of interest here in the case of interaction with the relatively light elements nitrogen and oxygen. Only Compton scattering would thus significantly contribute any photons. Heitler and Nordheim¹⁴ are credited as being the first to consider double Compton scattering and found the double cross section to be down from the single cross section by a factor of $1/137$. The error made by ignoring double scattering would thus seem to be acceptable. The fact that single scatter events dominate would seem to offer validity to the notion introduced here that the only significant scattering contributions are those coming from a cap region immediately below GRO.

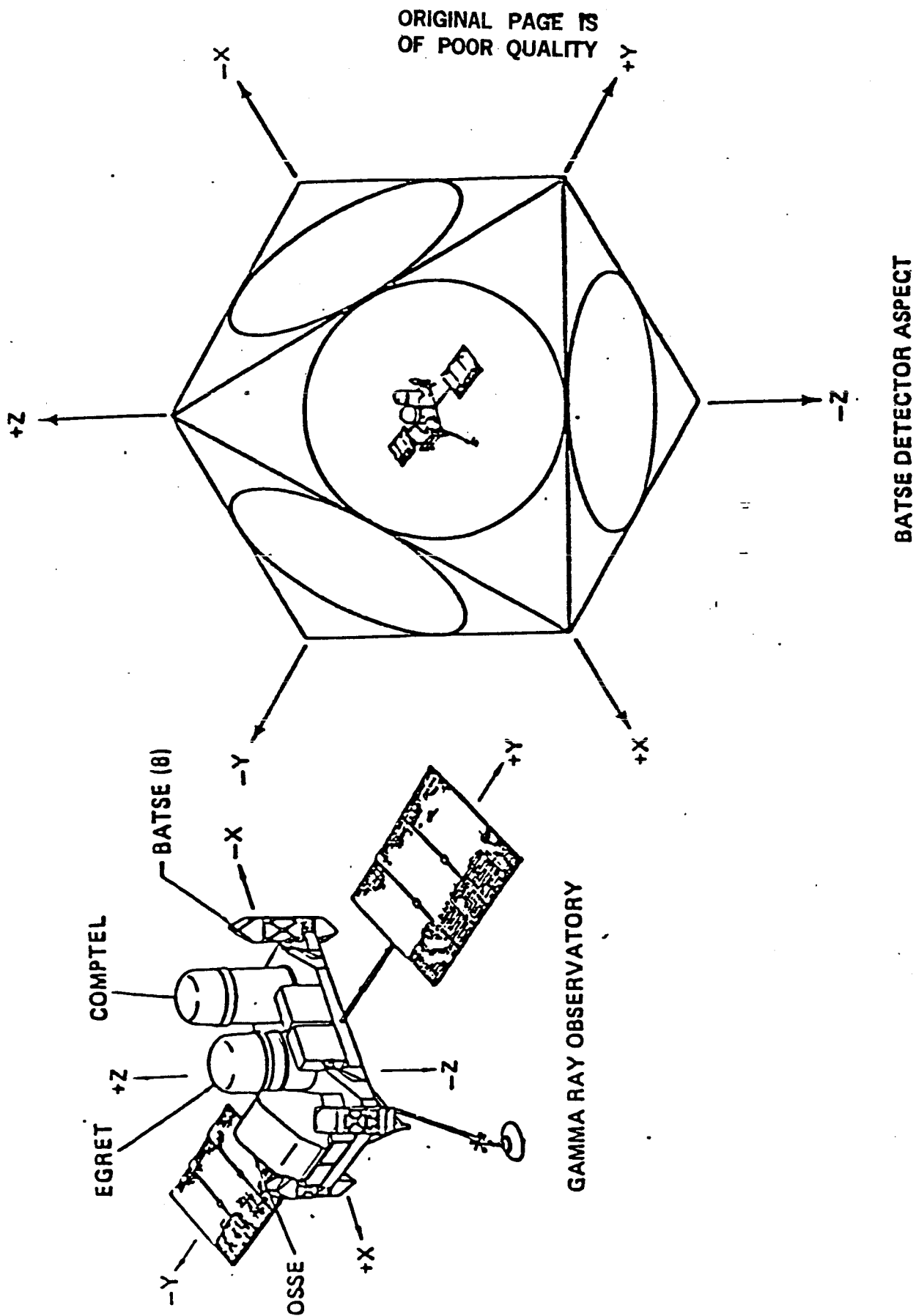


Figure 1. Gamma Ray Observatory Configuration; BATSE Octohedral Geometry

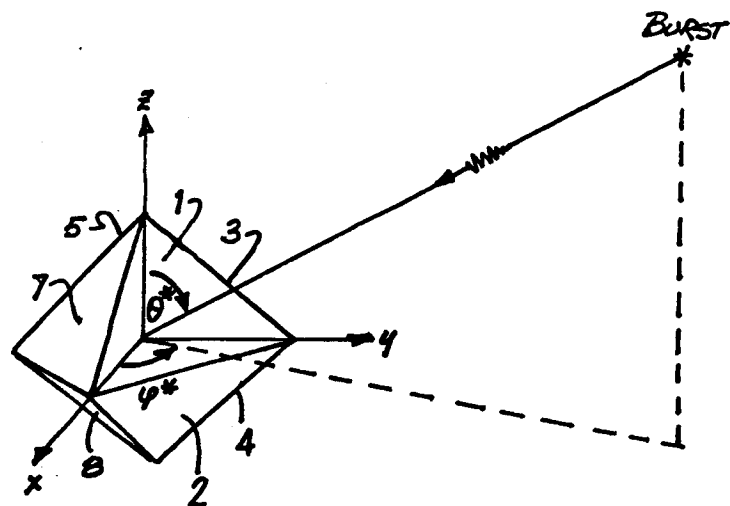


Figure 2. Burst Propagation Toward BATSE

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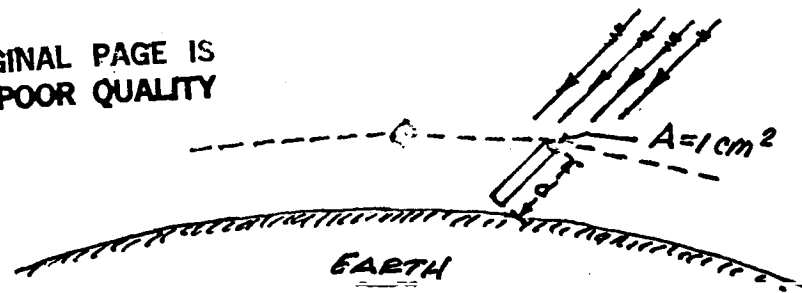


Figure 3. Photon Propagation Through a Column
of Atmosphere Below GRO

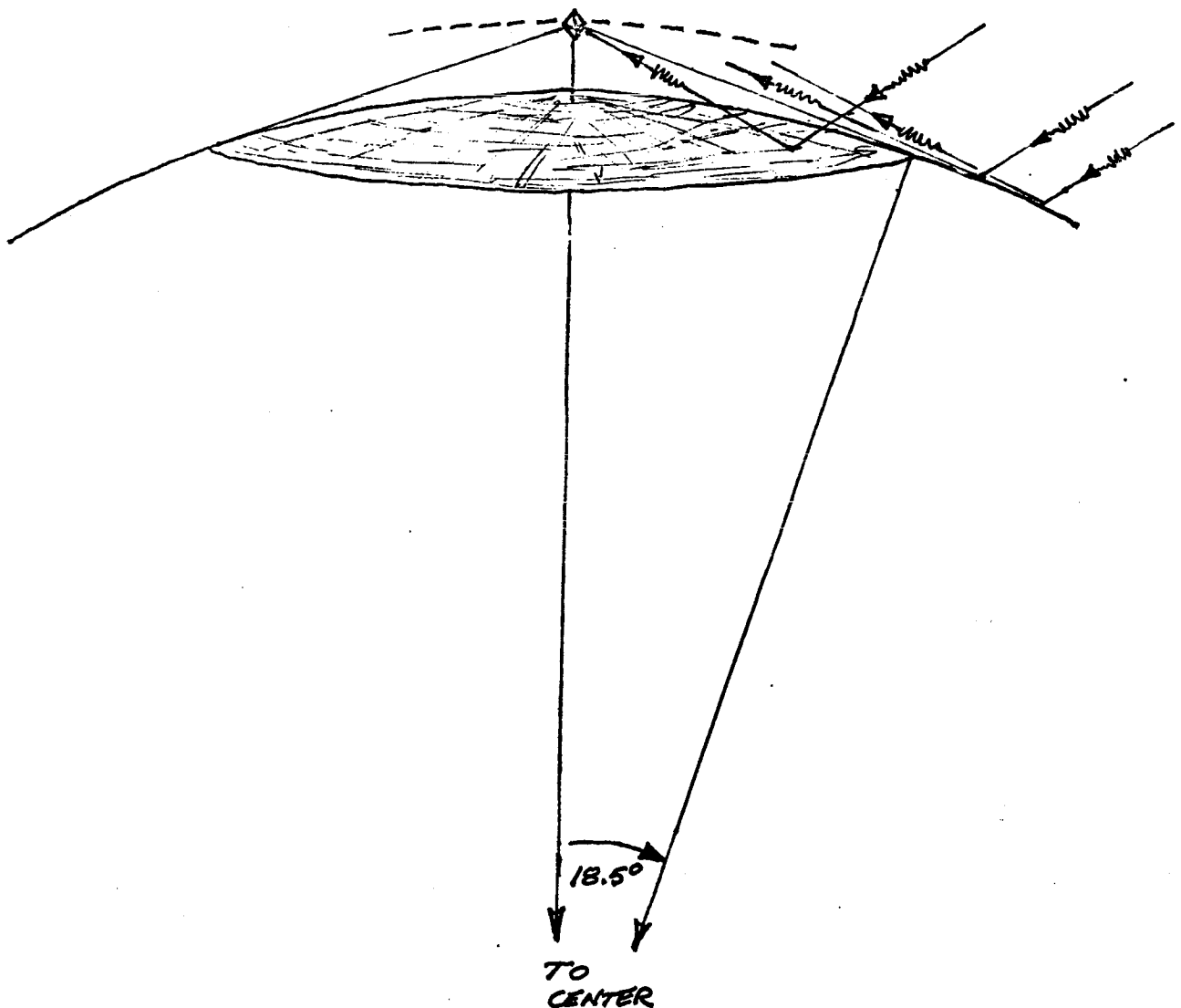


Figure 4. Single Scatter Region

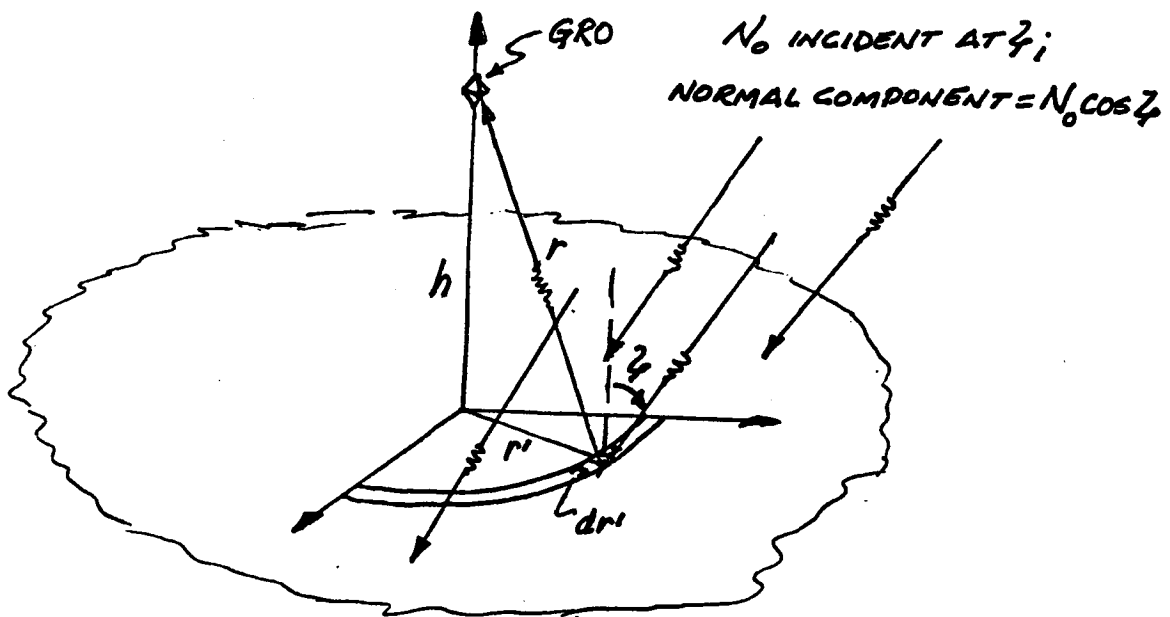


Figure 5-a. Disc Scattering into GRO

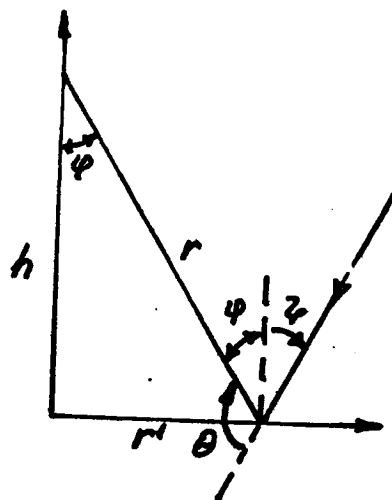


Figure 5-b. Geometry of Disc Scattering

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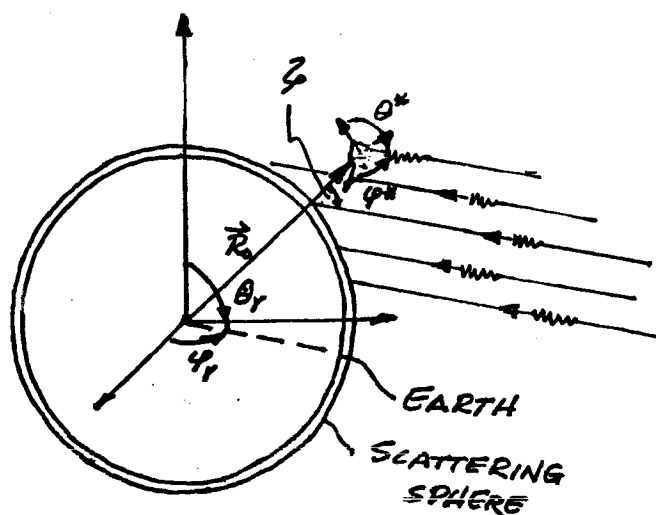


Figure 6. Schematic of Position Relationships

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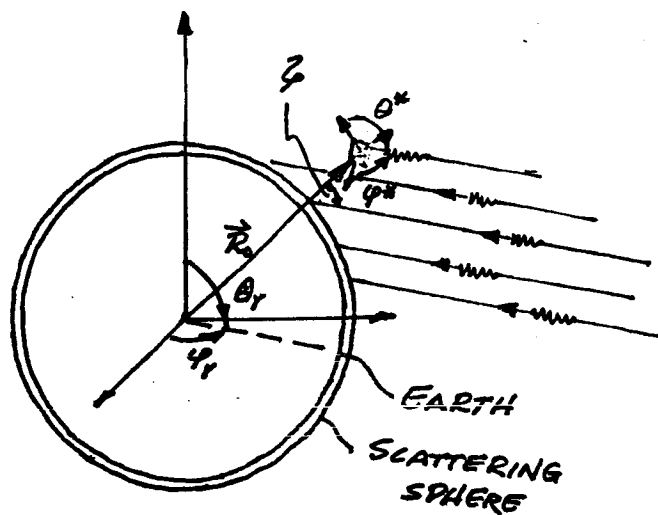


Figure 6. Schematic of Position Relationships

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